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Stability and Control

AE460 Aircraft Design

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Lecturer

Introduction

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- Complete Aircraft wing, tail and propulsion configuration, Mass Properties, including MOIs
- Non-Dimensional Derivatives (Roskam)
- Dimensional Derivatives (Etkin)
- Calculate System Matrix [A] and eigenvalues and eigenvectors
- Use results to determine stability (Etkin)

Reading:
Nicolai - CH 21, 22 & 23
Roskam – VI, CH 8 & 10

Other references:
MIL-STD-1797/MIL-F-8785 Flying Qualities of Piloted Aircraft
Airplane Flight Dynamics Part I (Roskam)

What are the requirements?

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Evaluate your aircraft for meeting the stability requirements
See SRD (Problem Statement) for values

- Flight Condition given:
  - Airspeed: M = ?
  - Altitude: ? ft.
  - Standard atmosphere
  - Configuration: ?
  - Fuel: ?%

- Longitudinal Stability:
  - $Cm_{CG\alpha} < 0$ at trim condition
  - Short period damping ratio: ?
  - Phugoid damping ratio: ?

- Directional Stability:
  - Dutch roll damping ratio: ?
  - Dutch roll undamped natural frequency: ?
  - Roll-mode time constant: ?
  - Spiral time to double amplitude: ?

Derivatives

- For General Equations of Unsteady Motions, reference Etkin, Chapter 4
- Assumptions
  - Aircraft configuration finalized
  - All mass properties are known, including MOI
  - Non-Dimensional Derivatives completed for flight condition analyzed
  - Aircraft is a rigid body
  - Symmetric aircraft across BL0, therefore $I_{xy}=I_{yz} = 0$
  - Axis of spinning rotors are fixed in the direction of the body axis and have constant angular speed
  - Assume a small disturbance
- Results in the simplified Linear Equations of Motion… on following slides
- But first: Convert non-dimensional derivatives to dimensional derivatives
  - Etkin, Tables 4.4 and 4.5 (also reference Table 4.1)
  - Takes flight path angle, reference areas/lengths, airspeed and altitude into account

Longitudinal Equation (Etkin Eq. 4.9,18)

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$$\dot{X} = \mathbf{A}\mathbf{x} + \Delta\mathbf{f}_c$$

$$\begin{bmatrix} \Delta\dot{u} \\ \dot{w} \\ \dot{q} \\ \Delta\dot{\theta} \end{bmatrix} = [4\ x\ 4\ Matrix] \begin{bmatrix} \Delta u \\ w \\ q \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} \frac{\Delta X_c}{m} \\ \frac{\Delta Z_c}{m - Z_{\dot{w}}} \\ \frac{\Delta M_c}{I_y} + \frac{M_{\dot{w}}}{I_y}\frac{\Delta Z_c}{m - Z_{\dot{w}}} \\ 0 \end{bmatrix}$$

Control forces, assume 0 for purposes of 460B

$\frac{X_u}{m}$	$\frac{X_w}{m}$	0	$-g\ \cos\theta_0$
$\frac{Z_u}{m - Z_{\dot{w}}}$	$\frac{Z_w}{m - Z_{\dot{w}}}$	$\frac{Z_q + mu_0}{m - Z_{\dot{w}}}$	$\frac{-mg\sin\theta_0}{m - Z_{\dot{w}}}$
$\frac{1}{I_y}\left[M_u + \frac{M_{\dot{w}}Z_u}{m - Z_{\dot{w}}}\right]$	$\frac{1}{I_y}\left[M_w + \frac{M_{\dot{w}}Z_w}{m - Z_{\dot{w}}}\right]$	$\frac{1}{I_y}\left[M_q + \frac{M_{\dot{w}}(Z_q + mu_0)}{m - Z_{\dot{w}}}\right]$	$\frac{-M_{\dot{w}}mg\sin\theta_0}{I_y(m - Z_{\dot{w}})}$
0	0	1	0

Eigenvalue/Eigenvector Review

$$\dot{X} = \mathbf{Ax} + \cancel{\Delta \mathbf{f}}_c$$

Solutions of the above first order differential equations are in the following form:

$$x(t) = x_0 e^{-\lambda t}$$

Eigenvector Eigenvalue

$$\lambda x_0 = \mathrm{A} x_0$$
$$(A - \lambda I) x_0 = 0$$
$$det(A - \lambda I) \quad = 0$$

Expansion of the above results in the Nth degree characteristic equations

Longitudinal Stability

MATLAB example (values from Etkin, 6.2,1)

Hint: Type command "format long" to obtain more decimal places

Command Window

Enter A Matrix

```
>> A=[-.006868 .01395 0 -32.2;-.09055 -.3151 773.98 0; .0001187 -.001026 -.4285 0; 0 0 1 0]

A =

   -0.0069    0.0140         0  -32.2000
   -0.0906   -0.3151  773.9800         0
    0.0001   -0.0010   -0.4285         0
         0         0    1.0000         0
```

Calculate eigenvalues and eigenvectors of Matrix A

```
>> [eigvec, eigval]=eig(A)

eigvec =

   0.0211 + 0.0166i   0.0211 - 0.0166i  -0.9983             -0.9983
   0.9996             0.9996            -0.0573 + 0.0097i  -0.0573 - 0.0097i
  -0.0001 + 0.0011i  -0.0001 - 0.0011i  -0.0001 - 0.0000i  -0.0001 + 0.0000i
   0.0011 - 0.0004i   0.0011 + 0.0004i   0.0001 + 0.0021i   0.0001 - 0.0021i


eigval =

  -0.3719 + 0.8875i        0                  0                  0
        0            -0.3719 - 0.8875i        0                  0
        0                  0            -0.0033 + 0.0672i        0
        0                  0                  0            -0.0033 - 0.0672i
```

Expand polynomial to obtain Characteristic Equation Coefficients

```
>> poly(eig(A))

ans =

    1.0000    0.7505    0.9355    0.0095    0.0042

fx >>
```

Longitudinal Stability

MATLAB example (values from Etkin, 6.2,1)

Mode 2 (short period mode)

$$\lambda_{3,4} = -.3719 \pm .8875i$$

```
eigval =

  -0.3719 + 0.8875i        0                      0                      0
        0              -0.3719 - 0.8875i          0                      0
        0                    0              -0.0033 + 0.0672i            0
        0                    0                      0              -0.0033 - 0.0672i

>> poly(eig(A))

ans =

    1.0000    0.7505    0.9355    0.0095    0.0042
```

Mode 1 (Phugoid mode)

$$\lambda_{1,2} = -.003289 \pm .06723i$$

$$\lambda^4 + .7505\lambda^3 + .9355\lambda^2 + .0095\lambda + .0042 = 0$$

Note: Characteristic Equation Roots contain real and imaginary roots
A negative real root means it is a stable mode

$$\lambda = n \pm \omega i$$

Quantitative Analysis

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- Period, $T = \frac{2\pi}{\omega}$
- Time to double (t_{double}) or time to half (t_{half}) $t_{double}\ or\ t_{half} = \frac{\ln(2)}{|\zeta|\omega_n}$
- Cycles to double (N_{double}) or cycles to half (N_{half}) $N_{double}\ or\ N_{half} = .110\frac{\sqrt{1-\zeta^2}}{|\zeta|}$

$$\lambda = n \pm \omega i$$

Mode 2 (short period mode) $\lambda_{3,4} = -.3719 \pm .8875i$

Mode 1 (Phugoid mode) $\lambda_{1,2} = -.003289 \pm .06723i$

$$\omega_n = \sqrt{\omega^2 + n^2}\text{, undamped circular frequency}$$

$$\zeta = \frac{-n}{\omega_n}\text{, damping ratio}$$

Lateral Equation (Etkin Eq. 4.9,19)

$$\begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = [4\ x\ 4\ Matrix] \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix} + \begin{bmatrix} \frac{\Delta Y_c}{m} \\ \frac{\Delta L_c}{I'_x} + I'_{zx} N_c \\ I'_{zx}\Delta L_c + \frac{\Delta N_c}{I'_z} \\ 0 \end{bmatrix}$$

Control forces, assume 0 for purposes of 460B

$\frac{Y_v}{m}$	$\frac{Y_p}{m}$	$\frac{Y_r}{m} - u_0$	$g\ \cos\theta_0$
$\frac{L_v}{I'_x} + I'_{zx}N_v$	$\frac{L_p}{I'_x} + I'_{zx}N_p$	$\frac{L_r}{I'_x} + I'_{zx}N_r$	0
$I'_{zx}L_v + \frac{N_v}{I'_z}$	$I'_{zx}L_p + \frac{N_p}{I'_z}$	$I'_{zx}L_r + \frac{N_r}{I'_z}$	0
0	1	$\tan\theta_0$	0

Lateral Equation (Etkin Eq. 4.9,19), continued

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Where:

$$I'_x = \left(\frac{I_x I_z - I_{zx}^2}{I_z}\right)$$

$$I'_z = \left(\frac{I_x I_z - I_{zx}^2}{I_x}\right)$$

$$I'_{zx} = \left(\frac{I_{zx}}{I_x I_z - I_{xz}^2}\right)$$

$\frac{Y_v}{m}$	$\frac{Y_p}{m}$	$\frac{Y_r}{m} - u_0$	$g\ \cos\theta_0$
$\frac{L_v}{I'_x} + I'_{zx} N_v$	$\frac{L_p}{I'_x} + I'_{zx} N_p$	$\frac{L_r}{I'_x} + I'_{zx} N_r$	0
$I'_{zx} L_v + \frac{N_v}{I'_z}$	$I'_{zx} L_p + \frac{N_p}{I'_z}$	$I'_{zx} L_r + \frac{N_r}{I'_z}$	0
0	1	$\tan\theta_0$	0

MATLAB example (values from Etkin, 6.7,1)

Hint: Type command "format long" to obtain more decimal places

```
B =

   -0.0558         0 -774.0000   32.2000
   -0.0039   -0.4342    0.4136         0
    0.0011   -0.0061   -0.1458         0
         0    1.0000         0         0
```

Enter B Matrix

```
>> [eigvec, eigval]=eig(B)

eigvec =

  -1.0000             -1.0000             -0.9972              0.9821
   0.0019 - 0.0032i    0.0019 + 0.0032i   -0.0367             -0.0014
  -0.0001 + 0.0011i   -0.0001 - 0.0011i    0.0021              0.0078
  -0.0035 - 0.0019i   -0.0035 + 0.0019i    0.0652              0.1880


eigval =

  -0.0330 + 0.9465i         0                   0                   0
        0             -0.0330 - 0.9465i         0                   0
        0                   0             -0.5625                   0
        0                   0                   0             -0.0073
```

Calculate eigenvalues and eigenvectors of Matrix B

```
>> poly(eig(B))

ans =

    1.0000    0.6358    0.9388    0.5114    0.0037
```

Expand polynomial to obtain Characteristic Equation Coefficients

MATLAB example (values from Etkin, 6.2,1)

Mode 3 (lateral oscillation or Dutch Roll)

$$\lambda_{3,4} = -.0330 \pm .9465i$$

```
eigval =

  -0.0330 + 0.9465i          0                     0              0
        0                -0.0330 - 0.9465i         0              0
        0                    0                 -0.5625            0
        0                    0                     0          -0.0073

>> poly(eig(B))

ans =

    1.0000    0.6358    0.9388    0.5114    0.0037
```

Mode 1 (Spiral mode)

$$\lambda_1 = -.0073$$

Mode 2 (Rolling Convergence)

$$\lambda_2 = -.5625$$

$$\lambda^4 + .6358\lambda^3 + .9388\lambda^2 + .5114\lambda + .0037 = 0$$

Note: Characteristic Equation Roots contain real and imaginary roots
A negative real root means it is a stable mode

$$\lambda = n \pm \omega i$$

Results

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Etkin Table 4.2

	Cx	Cz	Cm
u_hat	Cx_u	Cz_u	Cm_u
α	Cx_α	Cz_α	Cm_α
q_hat	Cx_q	Cz_q	Cm_q
α_dot_hat	$Cx_{\alpha dot}$	$Cz_{\alpha dot}$	$Cm_{\alpha dot}$

	Cx	Cz	Cm
u_hat	-0.108	-0.106	0.1043
α	0.2193	-4.92	-1.023
q_hat	0	-5.921	-23.92
α_dot_hat	0	5.896	-6.314

Etkin Table 4.3

	Cy	Cl	Cn
β	Cy_β	Cl_β	Cn_β
p_hat	Cy_p	Cl_p	Cn_p
r_hat	Cy_r	Cl_r	Cn_r
β_dot_hat	$Cy_{\beta dot}$	$Cl_{\beta dot}$	$Cn_{\beta dot}$

	Cy	Cl	Cn
β	-0.8771	-0.2797	0.1946
p_hat	0	-0.3295	-0.04073
r_hat	0	0.304	-0.2737
β_dot_hat	0	0	0

Etkin Table 4.4

	X	Z	M
u	X_u	Z_u	M_u
w	X_w	Z_w	M_w
q	X_q	Z_q	M_q
w_dot	X_{w_dot}	Z_{w_dot}	M_{w_dot}

	X	Z	M
u	-1.346E+02	-1.776E+03	3.551E+03
w	2.734E+02	-6.133E+03	-3.483E+04
q	0.000E+00	-1.008E+05	-1.112E+07
w_dot	0.000E+00	1.296E+02	-3.791E+03

Etkin Table 4.5

	Y	L	N
v	Y_v	L_v	N_v
p	Y_p	L_p	N_p
r	Y_r	L_r	N_r

	Y	L	N
v	-1.093E+03	-6.824E+04	4.747E+04
p	0.000E+00	-7.866E+06	-9.723E+05
r	0.000E+00	7.257E+06	-6.534E+06

Side Bar: Routh's Criteria for Stability

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Use to check for instability

Start with the characteristic equation

$$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0 \quad (A>0)$$

$$R = D(BC - AD) - B^2E > 0$$

Routh's discriminant

E = 0 and R = 0 are the boundaries between stability and instability

Assume a stable aircraft, and you change a design parameter resulting in an instability, then the following conditions hold:

- If only E changes from + to -, then one real root changes from – to +, that is, one divergence appears in the solution.
- If only R changes from + to -, then one real part of one complex pair of roots changes from – to +, that is, one divergent oscillation appears in the solution.

Stability Results

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Moments of Inertia CGref		
Ixx	1.83E+07	slugs-ft^2
Iyy	3.31E+07	slugs-ft^2
Izz	4.97E+07	slugs-ft^2
Izx	-1.56E+06	slugs-ft^2
Ixx'	1.83E+07	slugs-ft^2
Izz'	4.96E+07	slugs-ft^2
Izx'	-1.72E-09	slugs-ft^2

A (Longitudinal Equations, Etkin 4.9,18)			
-6.8040E-03	1.3816E-02	0.0000E+00	-32.174
-0.0904	-0.3120	774.3	0
1.1512E-04	-1.0165E-03	-4.2462E-01	0
0	0	1	0

B (Lateral Equations, Etkin 4.9,19)			
-5.526E-02	0.000E+00	-7.743E+02	32.174
-3.820E-03	-4.293E-01	4.089E-01	0
1.075E-03	-6.088E-03	-1.443E-01	0
0	1	0	0

Characteristic Equation Coefficients

A	B	C	D	E
1	0.750468	0.935494	0.009463	0.0041959

Characteristic Equation Coefficients

A	B	C	D	E
1	0.6358	0.9388	0.5114	0.003682

R	0.00419
E>0?	STABLE
R>0?	STABLE

R	0.0422
E>0?	STABLE
R>0?	STABLE

Stability Results

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- Cells in BLUE are the requirements to meet in the SRD (Problem Statement)

			real	imaginary	Stable/Unstable?	undamped circular frequency	damping ratio	Period	time to double or half	Cycles to double or half	Time Contant
	Mode	λ	n	ω		ω_n (rad/s)	ζ	T (s)	t (s)	N	τ_R (s)
Longitudinal	**1 (Phugoid)**	1,2	-0.003289	0.06723	**STABLE**	0.0673	0.04886	**93.5**	**210.7**	**2.25**	
	2 (Short Period)	3,4	-0.3719	0.8875	**STABLE**	0.9623	0.38648	**7.1**	**1.9**	**0.26**	
Lateral	**1 (Spiral)**	1	-0.0072973		**STABLE**	0.0073	1.00000	n/a	95.0	n/a	
	2 (Rolling Convergence)	2	-0.56248		**STABLE**	0.5625	1.00000	n/a	1.2	n/a	1.8
	3 (Lateral Oscillation/Dutch Roll)	3,4	-0.033011	0.94655	**STABLE**	0.9471	0.03485	6.6	21.0	3.15	

Note: τ_R (time constant) = $-1/\lambda_2$

Roll Control

(Aileron Sizing)

Roll Power

- For JT, use Class IVA per MIL-HDBK-1797
  - 90° in 1.3s
- For CAS, use Class IVC per MIL-HDBK-1797
  - 90° in 1.7s
- For SSBJ use 14CFR25 requirements

$$P = -\frac{2V}{b}\frac{C_{l_{\delta_\alpha}}}{C_{l_p}}\delta_\alpha$$ (Nicolai Eq 21.17b)

Roll Rate (rad/s)

Velocity (ft/s)

Wing span (ft)

Aileron control power (see R VI 10.3.5)

Roll dampening coefficient (see R VI Eq 10.51)

Aileron deflection

Pitch Control

(Elevator/Stabilator Sizing)

Pitch Sizing

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Fall Semester we used similar tail volume coefficients to size the horizontal stabilizer, but more analysis is required to ensure pitch control is sufficient and it is optimized for trim drag.

- Trim drag of horizontal stab should be <10% of overall aircraft drag (<5% for SSBJ)
- Only evaluate the drag due to lift (Cdo already accounted for)
- Too much Trim Drag?
  - Move CG aft closer to neutral point
  - Increase tail volume coefficient by increate tail area or moving aft
    - Ancillary benefit: both help to move CG aft
  - Increase tail aspect ratio to increase $C_{L\alpha T}$

Nicolai References (Student Exercises)

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- Trim Flight
  - Sections 22.2, 22.3 or 22.4

Side Bar: Wing incidence?

- Maneuver Flight
  - Pull up Maneuver: Sections 22.5, 22.6 or 22.7
  - Level Turn: Section 22.8

- Assumption: Assume +/- 20 degrees max deflection, but can go higher, up to 30 degrees with diminishing returns

Include in report, sizing for:

- Trim Flight sizing
- 7.5 g pull up (structural load factor)
- 5 g sustained turn
- Takeoff Rotation (rotate about MLG, see Nicolai Fig 23.3)
- Stretch goal: High AoA at low speed, i.e. landing (note: Ground effect increases stability)

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Yaw Control

(Rudder Sizing)

Rudder Power

Fall Semester we used similar tail volume coefficients to size the vertical stabilizer, but more analysis is required to ensure yaw control is sufficient.

- Requirements:
  - Crosswind landing
  - **OEI (this in the only one we are concerned about in 460B)**
  - Adverse yaw
    - Roll right cause left sideslip, requiring rudder input to correct (coordinated turn)
- Methodology already presented for OEI

Backup

Control Axis

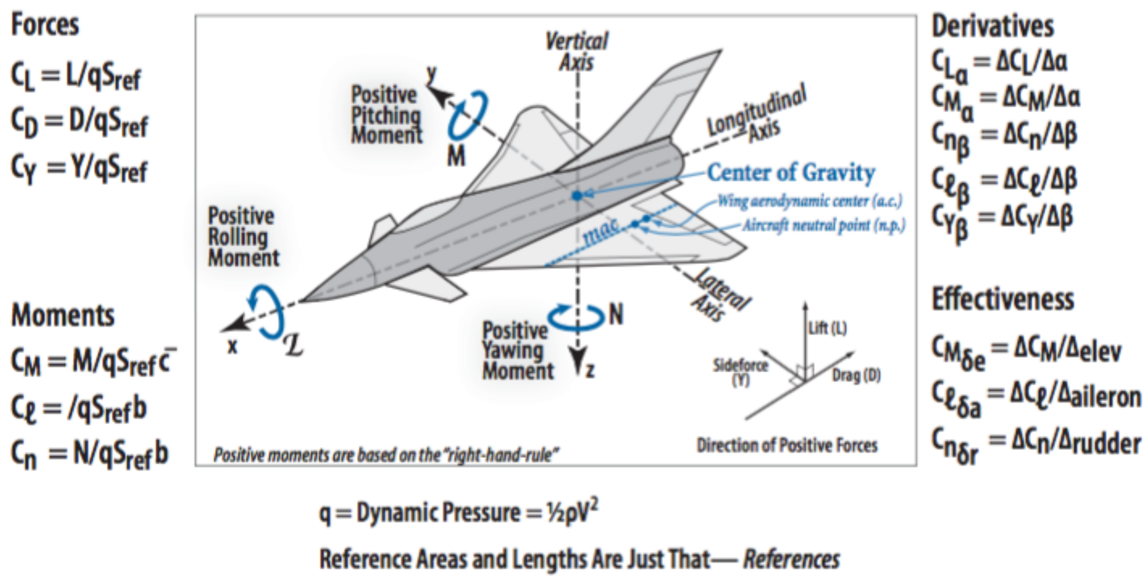

Figure 21.1 Major nondimensional aerodynamic parameters and sign convention. (Nicolai)

(Nicolai)

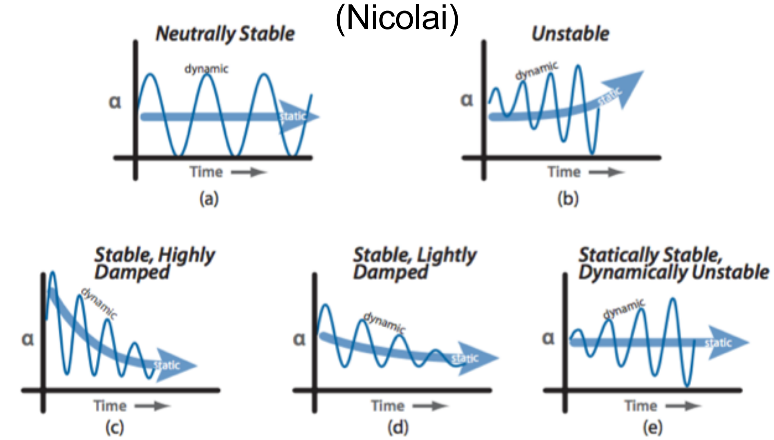

Figure 21.2 Static and dynamic stability about the pitch axis.

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