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Mass Properties w/MOI

a.k.a. Weight and Balance

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Lecturer

Introduction

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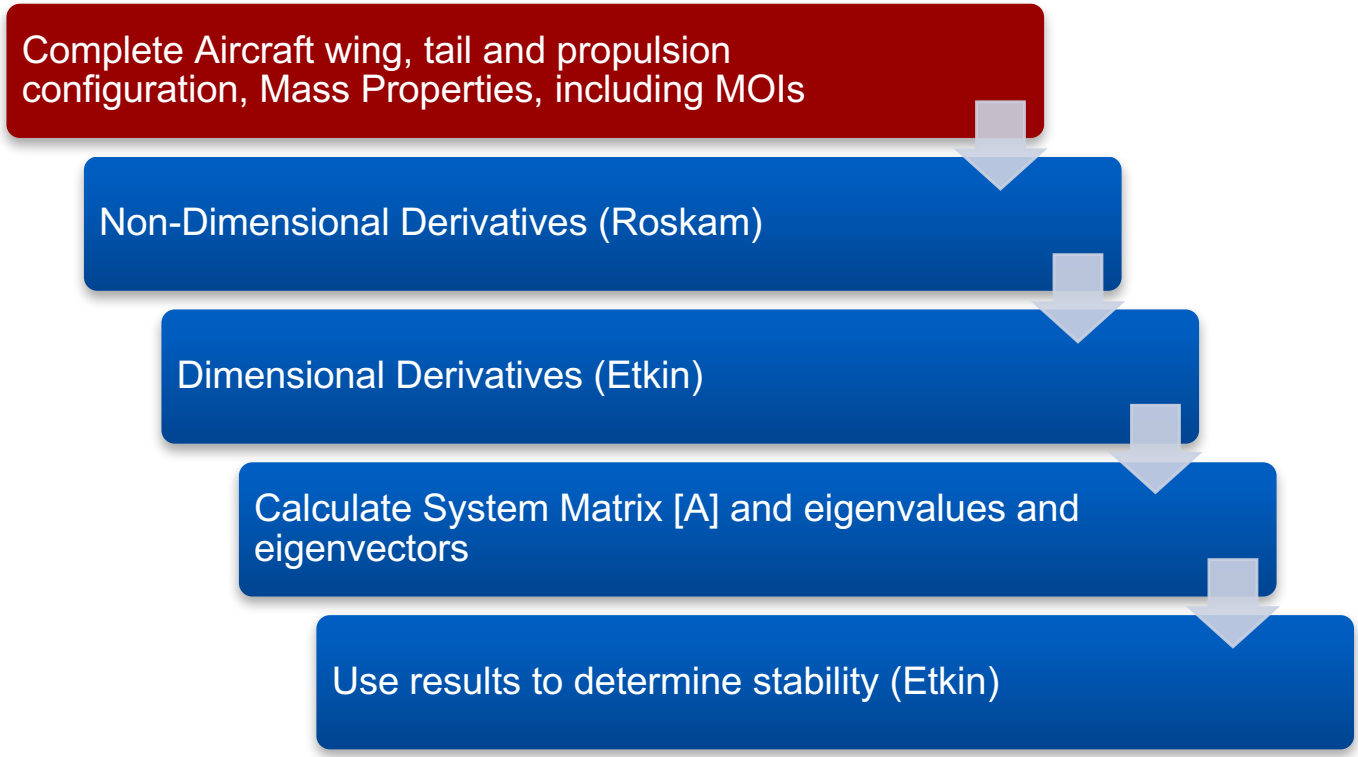

Reading:
Nicolai - CH 21, 22 & 23
Roskam – VI, CH 8 & 10

Other references:
MIL-STD-1797/MIL-F-8785 Flying Qualities of Piloted Aircraft
Airplane Flight Dynamics Part I (Roskam)

Background

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- Calculation of CG and Moments of Inertia (MOI) is an important input to the stability and control analysis (S&C) of an aircraft.
- MOI and Coordinate System is locked together, need to understand the coordinate system you are talking about, i.e.,
  - Coordinate system around the center of gravity of an object
  - ACS (0,0,0)
  - Coordinate system around the center of gravity of the aircraft
- CAD programs do a great job of Mass Properties analysis as long as the following rules are followed:
  - Parts are geometrically similar to the estimated final design
    - Note: a solid wing is not going to be the same as a wing with spars and ribs, even if the mass is the same.
  - The MOI output values being used for S&C are referenced to the correct coordinate system
  - Mass in the CAD model shall be corrected for the part being analyzed
    - Note: In preliminary design, it is acceptable to approximate the geometry of a part, and using an estimated weight, to “force” a correct weight of the part in CAD or “fudge” a fictitious density to obtain the estimated weight, to obtain a more accurate measure of MOI – more on this later.

Mass Properties Spreadsheet

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- Using a spreadsheet, all parts of an aircraft can be accounted for with respect to Weight (mass), CG location and Moments of Inertia.

Part Weight

CG of each part in REF CS (ACS 0, 0 , 0)

MOI of each part in REF CS (ACS 0,0,0)

Item	Name	Weight W	Distance X	Distance Y	Distance Z	Moment X	Moment Y	Moment Z	Radius of Gyration k_X	Radius of Gyration k_Y	Radius of Gyration k_Z	"I" from Ref CS I_{XX}	"I" from Ref CS I_{YY}	"I" from Ref CS I_{ZZ}	"I" from Ref CS I_{XZ}		Radius of Gyration Calculator R	r	h	k_X	k_Y	k_Z
		lb	in	in	in	lb-in	lb-in	lb-in	in	in	in	lb-in^2	lb-in^2	lb-in^2	lb-in^2		in	in	in	in	in	in
	Cylinder Shell	1614	300.000	0.000	80.000	484267	0	129138	17.981	70.519	70.519	10852952	163638498	153307468	38741365	Cylinder Shell	18.000	17.968	240.000	17.981	70.519	70.519
	Frame 1	236	200.000	0.000	80.000	47166	0	18866	12.705	8.984	8.984	1547369	10961461	9452161	3773251	Cylinder Solid	17.968		0.125	12.705	8.984	8.984
	Frame 2	236	210.000	0.000	80.000	49524	0	18866	12.705	8.984	8.984	1547369	11928356	10419056	3961913	Cylinder Solid	17.968		0.125	12.705	8.984	8.984
	Frame 3	236	220.000	0.000	80.000	51882	0	18866	12.705	8.984	8.984	1547369	12942418	11433117	4150576	Cylinder Solid	17.968		0.125	12.705	8.984	8.984
	Frame 4	236	230.000	0.000	80.000	54240	0	18866	12.705	8.984	8.984	1547369	14003644	12494344	4339238	Cylinder Solid	17.968		0.125	12.705	8.984	8.984
	Frame 5	236	240.000	0.000	80.000	56599	0	18866	12.705	8.984	8.984	1547369	15112037	13602736	4527901	Cylinder Solid	17.968		0.125	12.705	8.984	8.984
	Frame 6	236	250.000	0.000	80.000	58957	0	18866	12.705	8.984	8.984	1547369	16267595	14758294	4716563	Cylinder Solid	17.968		0.125	12.705	8.984	8.984
	Frame 7	236	260.000	0.000	80.000	61315	0	18866	12.705	8.984	8.984	1547369	17470318	15961018	4905226	Cylinder Solid	17.968		0.125	12.705	8.984	8.984
	Frame 8	236	270.000	0.000	80.000	63674	0	18866	12.705	8.984	8.984	1547369	18720207	17210907	5093888	Cylinder Solid	17.968		0.125	12.705	8.984	8.984
	Frame 9	236	280.000	0.000	80.000	66032	0	18866	12.705	8.984	8.984	1547369	20017262	18507962	5282551	Cylinder Solid	17.968		0.125	12.705	8.984	8.984
	Frame 10	236	290.000	0.000	80.000	68390	0	18866	12.705	8.984	8.984	1547369	21361483	19852183	5471213	Cylinder Solid	17.968		0.125	12.705	8.984	8.984
	Frame 11	236	300.000	0.000	80.000	70748	0	18866	12.705	8.984	8.984	1547369	22752869	21243569	5659876	Cylinder Solid	17.968		0.125	12.705	8.984	8.984
	Frame 12	236	310.000	0.000	80.000	73107	0	18866	12.705	8.984	8.984	1547369	24191421	22682120	5848538	Cylinder Solid	17.968		0.125	12.705	8.984	8.984
	Frame 13	236	320.000	0.000	80.000	75465	0	18866	12.705	8.984	8.984	1547369	25677138	24167838	6037201	Cylinder Solid	17.968		0.125	12.705	8.984	8.984
	Frame 14	236	330.000	0.000	80.000	77823	0	18866	12.705	8.984	8.984	1547369	27210021	25700721	6225863	Cylinder Solid	17.968		0.125	12.705	8.984	8.984
	Frame 15	236	340.000	0.000	80.000	80182	0	18866	12.705	8.984	8.984	1547369	28790070	27280769	6414526	Cylinder Solid	17.968		0.125	12.705	8.984	8.984
	Frame 16	236	350.000	0.000	80.000	82540	0	18866	12.705	8.984	8.984	1547369	30417284	28907984	6603188	Cylinder Solid	17.968		0.125	12.705	8.984	8.984
	Frame 17	236	360.000	0.000	80.000	84898	0	18866	12.705	8.984	8.984	1547369	32091664	30582364	6791851	Cylinder Solid	17.968		0.125	12.705	8.984	8.984
	Frame 18	236	370.000	0.000	80.000	87256	0	18866	12.705	8.984	8.984	1547369	33813209	32303909	6980513	Cylinder Solid	17.968		0.125	12.705	8.984	8.984
	Frame 19	236	380.000	0.000	80.000	89615	0	18866	12.705	8.984	8.984	1547369	35581921	34072620	7169176	Cylinder Solid	17.968		0.125	12.705	8.984	8.984
	Frame 20	236	390.000	0.000	80.000	91973	0	18866	12.705	8.984	8.984	1547369	37397797	35888497	7357838	Cylinder Solid	17.968		0.125	12.705	8.984	8.984
	Frame 21	236	400.000	0.000	80.000	94331	0	18866	12.705	8.984	8.984	1547369	39260840	37751540	7546501	Cylinder Solid	17.968		0.125	12.705	8.984	8.984

ΣW	6567	lb

Note: Format numbers as above, i.e. right justified, decimal point, etc.

X_{CG}	Y_{CG}	Z_{CG}
in	in	in
300	0	80

Total Weight

CG in REF CS

Ref CS

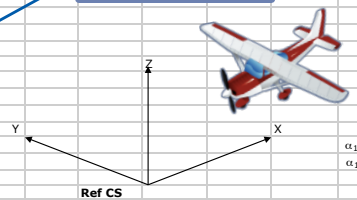
ΣI_{XX}	ΣI_{YY}	ΣI_{ZZ}	ΣI_{XZ}
lb-in^2	lb-in^2	lb-in^2	lb-in^2
43347694	659607512	617581177	157598756

Total MOI in REF CS (ACS 0,0,0)

Body CS at CG

ΣI_{XXB}	ΣI_{YYB}	ΣI_{ZZB}	ΣI_{XZB}
lb-in^2	lb-in^2	lb-in^2	lb-in^2
1321359	26585843	26585843	0

Total MOI at Body CS CG (300, 0, 80)



		Tranform Matrix			x	ΣI_{XXB} / ΣI_{ZZB} / ΣI_{XZB}	=	ΣI_{XXS} / ΣI_{ZZS} / ΣI_{XZS}
α_1 (deg)	0	1	0	0		1321359		1321359
α_1 (rad)	0	0	1	0	x	26585843	=	26585843
		0	0	1		0		0
α_1 (deg)	5.0	0.992	0.008	-0.174		1321359		1513271
α_1 (rad)	0.087	0.008	0.992	0.174	x	26585843	=	26393931
		0.087	-0.087	0.985		0		-2193566

Total MOI at CG, transformed to Stability Axis for AoA

Weight and CG

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Item	Name	Weight W	Distance X	Distance Y	Distance Z	Moment X	Moment Y	Moment Z	Radius of Gyration k_X	Radius of Gyration k_Y	Radius of Gyration k_Z	"I" from Ref CS I_{XX}	I_{YY}	I_{ZZ}	I_{XZ}
		lb	in	in	in	lb-in	lb-in	lb-in	in			$lb\text{-}in^2$	$lb\text{-}in^2$	$lb\text{-}in^2$	$lb\text{-}in^2$
	Cylinder Solid	6567	300.000	0.000	80.000	2E+06	0	525360	12.728						
	ΣW	6567	lb												

Radius of Gyration Calculator R	r	h	k_X	k_Y	k_Z

Step 1: Enter the part (Item) **Name**, **Weight** of part and its X, Y, Z **Distance** w.r.t REF CS (0,0,0).
Step 2: Spreadsheet Calculates the Moments and the CG location of all items entered, see math below:

Note: Format numbers as above, i.e. right justified, decimal point, etc.

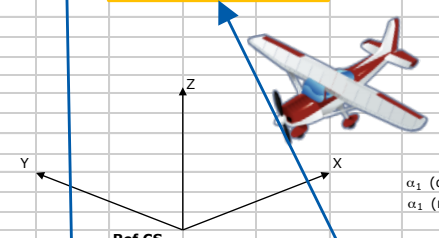
X_{CG}	Y_{CG}	Z_{CG}
in	in	in
300	0	80

Ref CS

ΣI_{XX}	ΣI_{YY}	ΣI_{ZZ}	ΣI_{XZ}
$lb\text{-}in^2$	$lb\text{-}in^2$	$lb\text{-}in^2$	$lb\text{-}in^2$
43092654	665112327	623083527	157608000

Body CS at CG

ΣI_{XXB}	ΣI_{YYB}	ΣI_{ZZB}	ΣI_{XZB}
$lb\text{-}in^2$	$lb\text{-}in^2$	$lb\text{-}in^2$	$lb\text{-}in^2$
1063854	32053527	32053527	0



		Tranform Matrix				ΣI_{XXB} ΣI_{ZZB} ΣI_{XZB}		ΣI_{XXS} ΣI_{ZZS} ΣI_{XZS}
α_1 (deg)	0	1	0	0		1063854		1063854
α_1 (rad)	0	0	1	0	x	32053527	=	32053527
		0	0	1		0		0
α_1 (deg)	5.0	0.992	0.008	-0.174		1063854		1299255
α_1 (rad)	0.087	0.008	0.992	0.174	x	32053527	=	31818126
		0.087	-0.087	0.985		0		-2690650

SUM of Weights

CG Calculation

$$X_{CG} = \frac{\sum_1^n X_{CG_n} W_n}{\sum_1^n W_n} \qquad Y_{CG} = \frac{\sum_1^n Y_{CG_n} W_n}{\sum_1^n W_n} \qquad Z_{CG} = \frac{\sum_1^n Z_{CG_n} W_n}{\sum_1^n W_n}$$

Calculate Radius Of Gyration

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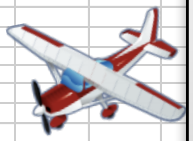
Item	Name	Weight W	Distance X	Distance Y	Distance Z	Moment X	Moment Y	Moment Z	Radius of Gyration k_X	Radius of Gyration k_Y	Radius of Gyration k_Z	"I" from Ref CS I_{XX}	"I" from Ref CS I_{YY}	"I" from Ref CS I_{ZZ}	"I" from Ref CS I_{XZ}
		lb	in	in	in	lb-in	lb-in	lb-in	in	in	in	lb-in^2	lb-in^2	lb-in^2	lb-in^2
	Cylinder Solid	6567	300.000	0.000	80.000	2E+06	0	525360	12.728	69.864	69.864	43092654	665112327	623083527	157608000
	ΣW	6567	lb		Note: Format numbers as above, i.e. right justified, decimal point, etc.										

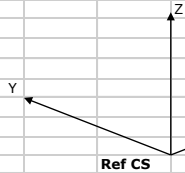
X_{CG}	Y_{CG}	Z_{CG}
in	in	in
300	0	80

Ref CS

ΣI_{XX}	ΣI_{YY}	ΣI_{ZZ}	ΣI_{XZ}
lb-in^2	lb-in^2	lb-in^2	lb-in^2

Cylinder Solid	Radius of Gyration Calculator R	r	h	k_X	k_Y	k_Z
	in	in	in	in	in	in
	18.000		240.000	12.728	69.864	69.864





Step 3: Calculate the Radius Of Gyration (k) for each component

Note: Each shape type will have a different formula, the example above is only for a SOLID CYLINDER

Step 4: You may choose other calculation methods, i.e. direct calculation of MOI, but k is convenient, since it is mass-independent.

Step 5: Enter Values in appropriate columns for k

Calculate MOI of each Part at REF CS

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Item	Name	Weight W	Distance X	Distance Y	Distance Z	Moment X	Moment Y	Moment Z	Radius of Gyration k_X	Radius of Gyration k_Y	Radius of Gyration k_Z	"I" from Ref CS I_{XX}	"I" from Ref CS I_{YY}	"I" from Ref CS I_{ZZ}	"I" from Ref CS I_{XZ}
		lb	in	in	in	lb-in	lb-in	lb-in	in	in	in	lb-in^2	lb-in^2	lb-in^2	lb-in^2
	Cylinder Solid	6567	300.000	0.000	80.000	2E+06	0	525360	12.728	69.864	69.864	43092654	665112327	623083527	157608000

Cylinder Solid	Radius of Gyration Calculator R	r	h	k_X	k_Y	k_Z
	in	in	in	in	in	in
	18.000		240.000	12.728	69.864	69.864

ΣW	6567	lb

Note: Format numbers as above, i.e. right justified, decimal point, etc.

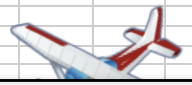
X_{CG}	Y_{CG}	Z_{CG}
in	in	in
300	0	80

Ref CS

ΣI_{XX}	ΣI_{YY}	ΣI_{ZZ}	ΣI_{XZ}
lb-in^2	lb-in^2	lb-in^2	lb-in^2
43092654	665112327	623083527	157608000

Body CS at CG

ΣI_{XXB}	ΣI_{YYB}	ΣI_{ZZB}	ΣI_{XZB}
lb-in^2	lb-in^2	lb-in^2	lb-in^2
1063854	32053527	32053527	0



Step 6: Using Radius Of Gyration (k), calculate the MOI values for each item referenced to Ref CS (0, 0, 0) using Parallel Axis Theorem, see below

Ref CS

α_1 (deg)	5.0	0.992	0.008	-0.174		1063854		1299255
α_1 (rad)	0.087	0.008	0.992	0.174	x	32053527	=	31818126
		0.087	-0.087	0.985		0		-2690650

$$I_{XX_R} = I_{XX_G} + W(y_G^2 + z_G^2) \quad I_{YY_R} = I_{YY_G} + W(x_G^2 + z_G^2) \quad I_{ZZ_R} = I_{ZZ_G} + W(x_G^2 + y_G^2)$$

$$I_{XX_R} = Wk_x^2 + W(y_G^2 + z_G^2) \quad I_{YY_R} = Wk_y^2 + W(x_G^2 + z_G^2) \quad I_{ZZ_R} = Wk_z^2 + W(x_G^2 + y_G^2)$$

$$\boldsymbol{I_{XX_R} = W\left(k_x^2 + y_G^2 + z_G^2\right)} \quad \boldsymbol{I_{YY_R} = W\left(k_y^2 + x_G^2 + z_G^2\right)} \quad \boldsymbol{I_{ZZ_R} = W\left(k_z^2 + x_G^2 + y_G^2\right)}$$

0, due to symmetry

$$\boldsymbol{I_{XZ_R} = I_{XZ_G} + W\,(z_g \times x_g)}$$

Transform MOI to Aircraft CG

Item	Name	Weight W (lb)	Distance X (in)	Distance Y (in)	Distance Z (in)	Moment X (lb-in)	Moment Y (lb-in)	Moment Z (lb-in)	Radius of Gyration k_X (in)	Radius of Gyration k_Y (in)	Radius of Gyration k_Z (in)	"I" from Ref CS I_{XX} (lb-in^2)	"I" from Ref CS I_{YY} (lb-in^2)	"I" from Ref CS I_{ZZ} (lb-in^2)	"I" from Ref CS I_{XZ} (lb-in^2)
	Cylinder Solid	6567	300.000	0.000	80.000	2E+06	0	525360	12.728	69.864	69.864	43092654	665112327	623083527	157608000

Cylinder Solid Radius of Gyration Calculator

R (in)	r (in)	h (in)	k_X (in)	k_Y (in)	k_Z (in)
18.000		240.000	12.728	69.864	69.864

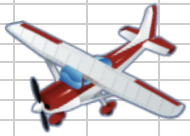
Step 7: Sum the MOI values for referenced to Ref CS (0, 0, 0)

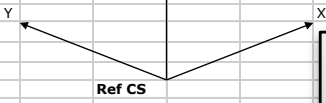
ΣW 6567 lb

Note: Format numbers as above, i.e. right justified, decimal point, etc.

X_{CG} (in)	Y_{CG} (in)	Z_{CG} (in)
300	0	80

Calculated CG of aircraft





Ref CS

ΣI_{XX} (lb-in^2)	ΣI_{YY} (lb-in^2)	ΣI_{ZZ} (lb-in^2)	ΣI_{XZ} (lb-in^2)
43092654	665112327	623083527	157608000

Body CS at CG

ΣI_{XXB} (lb-in^2)	ΣI_{YYB} (lb-in^2)	ΣI_{ZZB} (lb-in^2)	ΣI_{XZB} (lb-in^2)
1063854	32053527	32053527	0

Tranform Matrix	x	ΣI_{XXB} / ΣI_{ZZB} / ΣI_{XZB}	=	ΣI_{XXS} / ΣI_{ZZS} / ΣI_{XZS}

Step 8: Use Parallel to Transform the MOI Sum to the Body CS, located at the calculated CG of the aircraft.

MOI Calculation, to transform total MOI to Aircraft CG

$$I_{XX_B} = I_{XX_{CG}} = I_{XX_R} - \Sigma W \times (Y_{CG}^2 + Z_{CG}^2)$$

$$I_{YY_B} = I_{YY_{CG}} = I_{YY_R} - \Sigma W \times (X_{CG}^2 + Z_{CG}^2)$$

$$I_{ZZ_B} = I_{ZZ_{CG}} = I_{ZZ_R} - \Sigma W \times (X_{CG}^2 + Y_{CG}^2)$$

$$I_{XZ_B} = I_{XZ_{CG}} = I_{XZ_R} - \Sigma W (Z_{CG} \times X_{CG})$$

Transform MOI to Stability Control Axis

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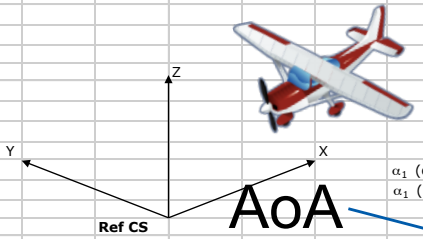
	Weight	Distance			Moment			Radius of Gyration			"I" from Ref CS				Radius of Gyration Calculator					
	W	X	Y	Z	X	Y	Z	k_X	k_Y	k_Z	I_{XX}	I_{YY}	I_{ZZ}	I_{XZ}	R	r	h	k_X	k_Y	k_Z
ΣW	6567	lb		Note: Format numbers as above, i.e. right justified, decimal point, etc.																

Step 9: There is no change in MOI for this step, but be aware, the Body axis at the CG requires to be rotated 180 degrees around YY Axis for use in the Stability Control Frame of Reference.

X_{CG}	Y_{CG}	Z_{CG}
in	in	in
300	0	80

Ref CS	ΣI_{XX}	ΣI_{YY}	ΣI_{ZZ}	ΣI_{XZ}
	lb-in^2	lb-in^2	lb-in^2	lb-in^2
	43092654	665112327	623083527	157608000

Body CS at CG	ΣI_{XXB}	ΣI_{YYB}	ΣI_{ZZB}	ΣI_{XZB}
	lb-in^2	lb-in^2	lb-in^2	lb-in^2
	1063854	32053527	32053527	0



		Tranform Matrix				ΣI_{XXB} / ΣI_{ZZB} / ΣI_{XZB}		ΣI_{XXS} / ΣI_{ZZS} / ΣI_{XZS}
α_1 (deg)	0	1	0	0	x	1063854	=	1063854
α_1 (rad)	0	0	1	0		32053527		32053527
		0	0	1		0		0
α_1 (deg)	5.0	0.992	0.008	-0.174		1063854		1299255
α_1 (rad)	0.087	0.008	0.992	0.174	x	32053527	=	31818126
		0.087	-0.087	0.985		0		-2690650

Step 10: Transform the Body Axis to the Stability Control Axis for the Specified AoA.

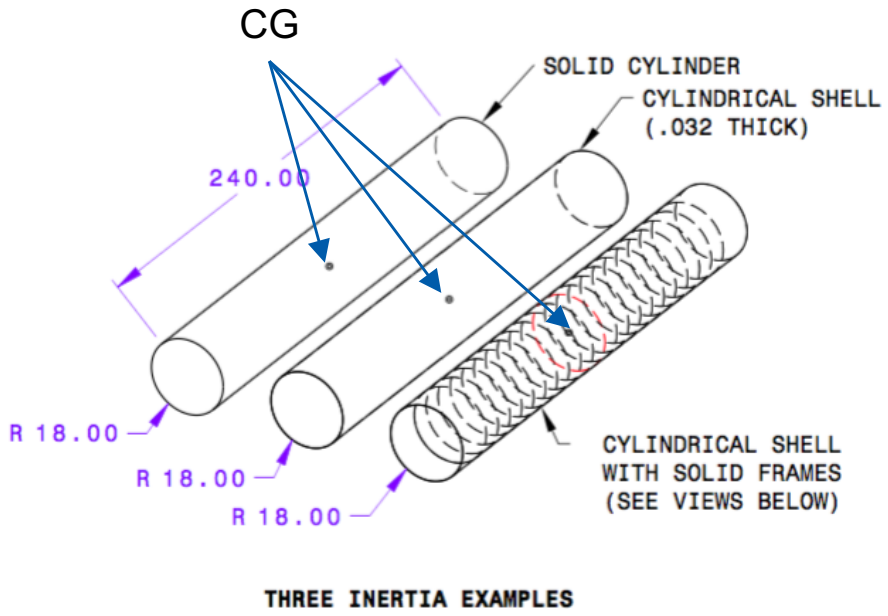
$$\begin{bmatrix} \cos^2\alpha & \sin^2\alpha & -\sin(2\alpha) \\ \sin^2\alpha & \cos^2\alpha & \sin(2\alpha) \\ \frac{1}{2}\sin(2\alpha) & -\frac{1}{2}\sin(2\alpha) & \cos 2\alpha \end{bmatrix} \times \begin{bmatrix} I_{XX_{CG}} \\ I_{ZZ_{CG}} \\ I_{XZ_{CG}} \end{bmatrix} = \begin{bmatrix} I_{XX_S} \\ I_{ZZ_S} \\ I_{XZ_S} \end{bmatrix}$$

$I_{YY_S} = I_{YY_{CG}}$ Since rotation is around YY axis

Where α = Angle of Attack at the specified Flight Condition

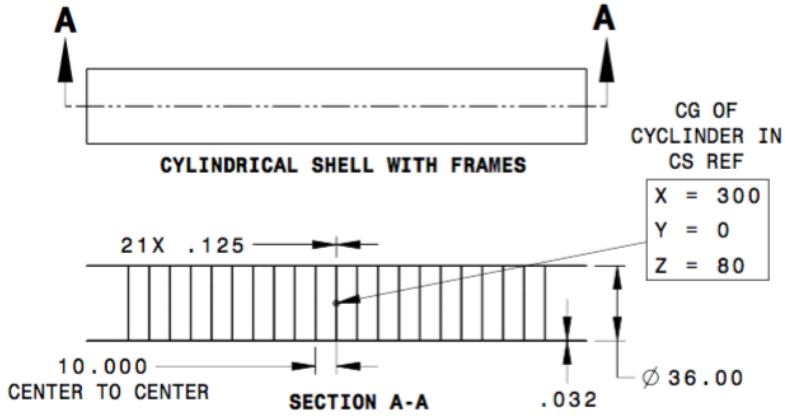
Example MOI of Various Cylindrical Objects

- Assumptions:
  - W = 6567 lbs of each configuration
  - Volume varies between configurations
  - Density is adjusted to achieve the same weight



THREE INERTIA EXAMPLES

CS REF of each respective cylinder



Example shows two methods, breaking down the cylinder with frames or use CAD for analysis

Solid Cylinder - Calculated vs. CAD

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Below is the calculated MOI vs. CAD analysis of a Solid Cylinder

				Ref CS	ΣI_{XX}	ΣI_{YY}	ΣI_{ZZ}	ΣI_{XZ}
					lb-in^2	lb-in^2	lb-in^2	lb-in^2
					43092654	665112327	623083527	157608000
				Body CS at CG	ΣI_{XXB}	ΣI_{YYB}	ΣI_{ZZB}	ΣI_{XZB}
					lb-in^2	lb-in^2	lb-in^2	lb-in^2
					1063854	32053527	32053527	0
		Tranform Matrix			x	ΣI_{XXB} ΣI_{ZZB} ΣI_{XZB}	=	ΣI_{XXS} ΣI_{ZZS} ΣI_{XZS}
α_1 (deg)	0	1	0	0		1063854		1063854
α_1 (rad)	0	0	1	0	x	32053527	=	32053527
		0	0	1		0		0
α_1 (deg)	5.0	0.992	0.008	#####		1063854		1299255
α_1 (rad)	0.087	0.008	0.992	0.174	x	32053527	=	31818126
		0.087	#####	0.985		0		-2690650

CAD Comparison

ΣI_{XX}	ΣI_{YY}	ΣI_{ZZ}	ΣI_{XZ}
lb-in^2	lb-in^2	lb-in^2	lb-in^2
43092718	665100000	623100000	-157600000

ΣI_{XXB}	ΣI_{YYB}	ΣI_{ZZB}	ΣI_{XZB}
lb-in^2	lb-in^2	lb-in^2	lb-in^2
1063856	32053575	32053575	0

ΣI_{XXS}	ΣI_{YYS}	ΣI_{ZZS}	ΣI_{XZS}
lb-in^2	lb-in^2	lb-in^2	lb-in^2
1299257	32053575	31818173	-2690654

This is confirmation the spreadsheet is correct, along with an understanding of the CAD tool.

Cylinder Shell - Calculated vs. CAD

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Below is the calculated MOI vs. CAD analysis of a Cylinder Shell

	ΣI_{XX}	ΣI_{YY}	ΣI_{ZZ}	ΣI_{XZ}
Ref CS	$lb\text{-}in^2$	$lb\text{-}in^2$	$lb\text{-}in^2$	$lb\text{-}in^2$
	44152087	665715743	623686943	157608000

	ΣI_{XXB}	ΣI_{YYB}	ΣI_{ZZB}	ΣI_{XZB}
Body CS at CG	$lb\text{-}in^2$	$lb\text{-}in^2$	$lb\text{-}in^2$	$lb\text{-}in^2$
	2123287	32656943	32656943	0

Tranform Matrix

						ΣI_{XXB}		ΣI_{XXS}
					x	ΣI_{ZZB}	=	ΣI_{ZZS}
						ΣI_{XZB}		ΣI_{XZS}
α_1 (deg)	0	1	0	0		2123287		2123287
α_1 (rad)	0	0	1	0	x	32656943	=	32656943
		0	0	1		0		0
α_1 (deg)	5.0	0.992	0.008	#####		2123287		2355225
α_1 (rad)	0.087	0.008	0.992	0.174	x	32656943	=	32425005
		0.087	#####	0.985		0		-2651057

CAD Comparison

ΣI_{XX}	ΣI_{YY}	ΣI_{ZZ}	ΣI_{XZ}
$lb\text{-}in^2$	$lb\text{-}in^2$	$lb\text{-}in^2$	$lb\text{-}in^2$
44152781	665600000	623600000	-157600000

ΣI_{XXB}	ΣI_{YYB}	ΣI_{ZZB}	ΣI_{XZB}
$lb\text{-}in^2$	$lb\text{-}in^2$	$lb\text{-}in^2$	$lb\text{-}in^2$
2123931	32583603	32583603	0

ΣI_{XXS}	ΣI_{YYS}	ΣI_{ZZS}	ΣI_{XZS}
$lb\text{-}in^2$	$lb\text{-}in^2$	$lb\text{-}in^2$	$lb\text{-}in^2$
2355307	32583603	32352227	-2644633

This is confirmation the spreadsheet is correct, along with an understanding of the CAD tool.

Cylinder Shell with Frames- Calculated vs. CAD

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Below is the calculated MOI vs. CAD analysis of a Cylinder Shell frames

										CAD Comparison			
				Ref CS	ΣI_{XX}	ΣI_{YY}	ΣI_{ZZ}	ΣI_{XZ}		ΣI_{XX}	ΣI_{YY}	ΣI_{ZZ}	ΣI_{XZ}
					lb-in^2	lb-in^2	lb-in^2	lb-in^2		lb-in^2	lb-in^2	lb-in^2	lb-in^2
					43347694	659607512	617581177	157598756		43350426	659600000	617600000	-157600000
				Body CS at CG	ΣI_{XXB}	ΣI_{YYB}	ΣI_{ZZB}	ΣI_{XZB}		ΣI_{XXB}	ΣI_{YYB}	ΣI_{ZZB}	ΣI_{XZB}
					lb-in^2	lb-in^2	lb-in^2	lb-in^2		lb-in^2	lb-in^2	lb-in^2	lb-in^2
					1321359	26585843	26585843	0		1321595	26569384	26569384	0
		Tranform Matrix				x	ΣI_{XXB} ΣI_{ZZB} ΣI_{XZB}		=	ΣI_{XXS} ΣI_{ZZS} ΣI_{XZS}			
α_1 (deg)	0	1	0	0			1321359			1321359			
α_1 (rad)	0	0	1	0		x	26585843		=	26585843			
		0	0	1			0			0			
α_1 (deg)	5.0	0.992	0.008	-0.174			1321359			1513271			
α_1 (rad)	0.087	0.008	0.992	0.174		x	26585843		=	26393931			
		0.087	-0.087	0.985			0			-2193566			

ΣI_{XXS}	ΣI_{YYS}	ΣI_{ZZS}	ΣI_{XZS}
lb-in^2	lb-in^2	lb-in^2	lb-in^2
1513381	26569384	26377598	-2192116

This is confirmation the spreadsheet is correct, along with an understanding of the CAD tool.

Mass Properties Spreadsheet Of Cylinder Shell with Frames

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Use this as a check of your spreadsheet to verify its working properly

Item	Name	Weight W lb	Distance X in	Distance Y in	Distance Z in	Moment X lb-in	Moment Y lb-in	Moment Z lb-in	Radius of Gyration k_X in	k_Y in	k_Z in	"I" from Ref CS I_{XX} lb-in^2	I_{YY} lb-in^2	I_{ZZ} lb-in^2	I_{XZ} lb-in^2		Radius of Gyration Calculator R in	r in	h in	k_X in	k_Y in	k_Z in
	Cylinder Shell	1614	300.000	0.000	80.000	484267	0	129138	17.981	70.519	70.519	10852952	163638498	153307468	38741365	**Cylinder Shell**	18.000	17.968	240.000	17.981	70.519	70.519
	Frame 1	236	200.000	0.000	80.000	47166	0	18866	12.705	8.984	8.984	1547369	10961461	9452161	3773251	**Cylinder Solid**	17.968		0.125	12.705	8.984	8.984
	Frame 2	236	210.000	0.000	80.000	49524	0	18866	12.705	8.984	8.984	1547369	11928356	10419056	3961913	**Cylinder Solid**	17.968		0.125	12.705	8.984	8.984
	Frame 3	236	220.000	0.000	80.000	51882	0	18866	12.705	8.984	8.984	1547369	12942418	11433117	4150576	**Cylinder Solid**	17.968		0.125	12.705	8.984	8.984
	Frame 4	236	230.000	0.000	80.000	54240	0	18866	12.705	8.984	8.984	1547369	14003644	12494344	4339238	**Cylinder Solid**	17.968		0.125	12.705	8.984	8.984
	Frame 5	236	240.000	0.000	80.000	56599	0	18866	12.705	8.984	8.984	1547369	15112037	13602736	4527901	**Cylinder Solid**	17.968		0.125	12.705	8.984	8.984
	Frame 6	236	250.000	0.000	80.000	58957	0	18866	12.705	8.984	8.984	1547369	16267595	14758294	4716563	**Cylinder Solid**	17.968		0.125	12.705	8.984	8.984
	Frame 7	236	260.000	0.000	80.000	61315	0	18866	12.705	8.984	8.984	1547369	17470318	15961018	4905226	**Cylinder Solid**	17.968		0.125	12.705	8.984	8.984
	Frame 8	236	270.000	0.000	80.000	63674	0	18866	12.705	8.984	8.984	1547369	18720207	17210907	5093888	**Cylinder Solid**	17.968		0.125	12.705	8.984	8.984
	Frame 9	236	280.000	0.000	80.000	66032	0	18866	12.705	8.984	8.984	1547369	20017262	18507962	5282551	**Cylinder Solid**	17.968		0.125	12.705	8.984	8.984
	Frame 10	236	290.000	0.000	80.000	68390	0	18866	12.705	8.984	8.984	1547369	21361483	19852183	5471213	**Cylinder Solid**	17.968		0.125	12.705	8.984	8.984
	Frame 11	236	300.000	0.000	80.000	70748	0	18866	12.705	8.984	8.984	1547369	22752869	21243569	5659876	**Cylinder Solid**	17.968		0.125	12.705	8.984	8.984
	Frame 12	236	310.000	0.000	80.000	73107	0	18866	12.705	8.984	8.984	1547369	24191421	22682120	5848538	**Cylinder Solid**	17.968		0.125	12.705	8.984	8.984
	Frame 13	236	320.000	0.000	80.000	75465	0	18866	12.705	8.984	8.984	1547369	25677138	24167838	6037201	**Cylinder Solid**	17.968		0.125	12.705	8.984	8.984
	Frame 14	236	330.000	0.000	80.000	77823	0	18866	12.705	8.984	8.984	1547369	27210021	25700721	6225863	**Cylinder Solid**	17.968		0.125	12.705	8.984	8.984
	Frame 15	236	340.000	0.000	80.000	80182	0	18866	12.705	8.984	8.984	1547369	28790070	27280769	6414526	**Cylinder Solid**	17.968		0.125	12.705	8.984	8.984
	Frame 16	236	350.000	0.000	80.000	82540	0	18866	12.705	8.984	8.984	1547369	30417284	28907984	6603188	**Cylinder Solid**	17.968		0.125	12.705	8.984	8.984
	Frame 17	236	360.000	0.000	80.000	84898	0	18866	12.705	8.984	8.984	1547369	32091664	30582364	6791851	**Cylinder Solid**	17.968		0.125	12.705	8.984	8.984
	Frame 18	236	370.000	0.000	80.000	87256	0	18866	12.705	8.984	8.984	1547369	33813209	32303909	6980513	**Cylinder Solid**	17.968		0.125	12.705	8.984	8.984
	Frame 19	236	380.000	0.000	80.000	89615	0	18866	12.705	8.984	8.984	1547369	35581921	34072620	7169176	**Cylinder Solid**	17.968		0.125	12.705	8.984	8.984
	Frame 20	236	390.000	0.000	80.000	91973	0	18866	12.705	8.984	8.984	1547369	37397797	35888497	7357838	**Cylinder Solid**	17.968		0.125	12.705	8.984	8.984
	Frame 21	236	400.000	0.000	80.000	94331	0	18866	12.705	8.984	8.984	1547369	39260840	37751540	7546501	**Cylinder Solid**	17.968		0.125	12.705	8.984	8.984
	ΣW	6567	lb		**Note: Format numbers as above, i.e. right justified, decimal point, etc.**																	

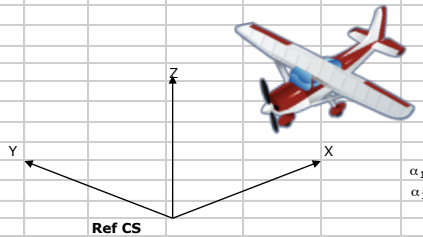
X_{CG} in	Y_{CG} in	Z_{CG} in
300	0	80

Ref CS

ΣI_{XX} lb-in^2	ΣI_{YY} lb-in^2	ΣI_{ZZ} lb-in^2	ΣI_{XZ} lb-in^2
43347694	659607512	617581177	157598756

Body CS at CG

ΣI_{XXB} lb-in^2	ΣI_{YYB} lb-in^2	ΣI_{ZZB} lb-in^2	ΣI_{XZB} lb-in^2
1321359	26585843	26585843	0



Tranform Matrix

						ΣI_{XXB}		ΣI_{XXS}
						ΣI_{ZZB}		ΣI_{ZZS}
						ΣI_{XZB}		ΣI_{XZS}
α_1 (deg)	0	1	0	0		1321359		1321359
α_1 (rad)	0	0	1	0	x	26585843	=	26585843
		0	0	1		0		0
α_1 (deg)	5.0	0.992	0.008	-0.174		1321359		1513271
α_1 (rad)	0.087	0.008	0.992	0.174	x	26585843	=	26393931
		0.087	-0.087	0.985		0		-2193566

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